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Application of CPM and PERT Methods in Residential Construction Project Scheduling: A Case Study

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Abstract

Effective project scheduling plays an important role in ensuring the timely completion of residential construction projects while controlling project costs and managing schedule uncertainty. This study aims to analyze the scheduling performance of a residential construction project by integrating the Critical Path Method (CPM), project crashing, and the Program Evaluation and Review Technique (PERT). The research employed a quantitative case study approach using activity duration, precedence relationships, and project cost data obtained from a residential construction project. CPM was applied to determine the critical path and normal project duration, crashing analysis was conducted to identify the most economical schedule acceleration strategy, and PERT was used to evaluate project completion uncertainty through probabilistic time estimation. The CPM analysis identified a normal project duration of 59 days with two critical paths controlling project completion. The crashing analysis reduced the project duration to 55 days through selective acceleration of critical activities with minimum additional cost. Furthermore, the PERT analysis produced an expected project duration of 43.833 days, a project standard deviation of 1.951 days, and a 99.60% probability of completing the project within the specified target duration. These findings demonstrate that integrating CPM, crashing analysis, and PERT provides a comprehensive framework for determining project schedules, evaluating cost-effective acceleration alternatives, and assessing project completion uncertainty. The proposed approach can support project managers in improving scheduling decisions for residential construction projects.

Keywords: *CPM; Construction Management; Crashing; PERT; Project Scheduling*

1. Introduction

The rapid growth of the construction industry has significantly increased the complexity of project implementation, both technically and managerially. Construction projects typically involve numerous interdependent activities, limited resources, and strict completion deadlines. Consequently, project success is no longer evaluated solely based on the quality of the completed work but also on the ability to complete the project within the planned schedule and budget. In practice, however, construction projects are frequently affected by various uncertainties, including delays in material procurement, changes in site conditions, fluctuating labor productivity, design modifications, and adverse weather conditions. These factors often result in schedule overruns, increased project costs, and reduced

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overall project performance. Therefore, effective project scheduling has become one of the fundamental components of construction project management to ensure timely and efficient project execution [1], [2], [3].

Project scheduling serves not only as a tool for determining the sequence of construction activities but also as an essential mechanism for allocating resources, monitoring project progress, and evaluating project performance throughout the construction process [4]. Over the years, numerous scheduling techniques have been developed to support project planning and control. Among these techniques, the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT) remain the most widely adopted approaches because of their relatively simple concepts, practical implementation, and proven usefulness in construction project scheduling [5].

The Critical Path Method (CPM) is a deterministic scheduling technique that assumes activity durations are known with certainty. The primary objective of CPM is to identify the critical path, namely the sequence of activities that directly determines the overall project duration. In addition, CPM provides information regarding Early Start (ES), Early Finish (EF), Late Start (LS), Late Finish (LF), and activity float, enabling project managers to identify critical activities requiring close supervision while utilizing available resources more efficiently. Nevertheless, the deterministic assumption underlying CPM limits its capability to represent the uncertainty that commonly occurs in real construction projects, where activity durations are often influenced by unpredictable external factors [6], [7], [8].

To address this limitation, the Program Evaluation and Review Technique (PERT) incorporates uncertainty into project scheduling by representing each activity duration using three time estimates: optimistic time, most likely time, and pessimistic time. These estimates are combined to calculate the expected activity duration and its associated variance, allowing project managers to estimate the probability of completing a project within a specified deadline. Consequently, PERT provides additional decision-support information by explicitly considering uncertainty in project scheduling. However, although PERT effectively evaluates schedule uncertainty, it does not directly assess project acceleration alternatives or the additional costs associated with schedule compression [7], [8].

Numerous studies have investigated the application of CPM and PERT in construction project scheduling. Bagshaw [5] demonstrated that CPM and PERT are effective tools for project planning and control through critical-path identification and project duration estimation. Ridwan [8] integrated CPM and PERT to improve project time management, while Fadjar et al. [8] employed PERT to estimate the completion time of residential construction projects using probabilistic activity durations. Other researchers have applied CPM and PERT to highway construction, building development, and pipeline installation projects with the primary objective of determining critical activities and estimating project completion times [9], [10], [11], [12], [13]. Furthermore, recent studies have reviewed the advantages, limitations, and future development of CPM and PERT for modern construction scheduling applications [14]. These studies consistently indicate that CPM and PERT contribute significantly to project planning and scheduling. Nevertheless, most previous studies have primarily focused on critical-path identification or project duration estimation independently rather than evaluating multiple scheduling aspects simultaneously.

Based on the existing literature, an important research gap can be identified. Most previous studies employed CPM to determine the critical path and PERT to estimate project duration under uncertainty. However, relatively limited attention has been given to integrating critical-path analysis, project acceleration through crashing, time–cost trade-off evaluation, and probabilistic completion analysis within a single analytical framework, particularly for residential construction projects. In practice, project managers require not only information regarding critical activities but also practical guidance on selecting the most economical schedule acceleration strategy, estimating the additional costs resulting from project compression, and evaluating the probability of completing the project within the desired schedule. Considering these aspects simultaneously provides a more comprehensive basis for project scheduling decisions than relying exclusively on either deterministic or probabilistic analysis [7], [15].

Accordingly, this study applies an integrated analytical framework consisting of the CPM, crashing analysis, and the PERT to evaluate the scheduling performance of a residential construction project in Bandar District, Simalungun Regency, Indonesia. CPM is employed to identify the project's critical path and initial completion time, crashing analysis is used to evaluate feasible schedule acceleration alternatives together with their additional costs, and PERT is applied to estimate the expected project duration and the probability of completing the project within a specified target time.

The contribution of this study does not lie in proposing a new scheduling algorithm. Instead, this study demonstrates the application of an integrated analytical framework that combines deterministic scheduling using CPM, time–cost trade-off analysis through crashing, and probabilistic schedule evaluation using

PERT within a single residential construction case study. By integrating these complementary approaches, the proposed framework provides more comprehensive information regarding critical activities, feasible acceleration alternatives, additional project costs, and schedule reliability, thereby supporting more informed decision-making in residential construction project scheduling.

2. Methods

2.1 The Research Design

This study employed a quantitative case-study approach to evaluate the scheduling performance of a residential construction project located in Bandar District, Simalungun Regency, North Sumatra, Indonesia. The research focused on analyzing the project schedule using three complementary analytical techniques: the Critical Path Method (CPM), project crashing analysis, and the Program Evaluation and Review Technique (PERT).

The analysis was conducted sequentially. First, CPM was applied to determine the project network, identify the critical path, and estimate the normal project duration. Second, crashing analysis was performed to investigate feasible schedule acceleration alternatives while considering the additional costs required for reducing activity durations. Finally, PERT was employed to evaluate the uncertainty associated with project completion by estimating the expected project duration and the probability of completing the project within a specified target time

2.2 Project Network Model

The project is represented as a directed graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$, where: $\boldsymbol{V}$ is the set of nodes (events), $\boldsymbol{E}$ is the set of activities (edges). Each activity $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{E}$ has a duration $\boldsymbol{t_{ij}}$ and specific dependency relationships. This model is used to represent the structure of the project network.

2.3 CPM Formulation

The CPM method is used to determine the start and finish times of each activity through forward pass and backward pass calculations [16].

The notations used are as follows:

t	=	activity duration
i	=	current job
j	=	next job
ICE	=	earliest start time
EF	=	earliest finish time
LF	=	latest finish time
LS	=	latest start time
R_t	=	time difference
F	=	free time
t_i	=	duration of activity i
a_i	=	optimistic time of activity i
m_i	=	the most likely time for activity i to occur
b_i	=	pessimistic time of activity i
W_n	=	acceleration of work duration from normal time
W_c	=	acceleration time
B_n	=	Normal costs
B_c	=	Crash costs
EET	=	Earliest Event Time
LET	=	Latest Event Time
σ_i^2	=	Variance of activity duration i
σ_i	=	Standard deviation
b_i	=	Pessimistic duration
a_i	=	Optimistic duration
z	=	Z-score or normal standard deviation
x	=	Target time duration
µ	=	Expected time duration
s	=	Critical path standard deviation

The CPM procedure includes the following calculations:

1. The earliest start time is determined by the maximum earliest finish time of all immediate predecessors:

$$ES_t = max\ (EF)_k \tag{1}$$

2. The earliest finish time is calculated as:

$$EF_t = ES_t + t \tag{2}$$

3. The latest start time is given by:

$$LS_t = LF_t - t \tag{3}$$

4. The total float is defined as:

$$R_t = LF_t - ES_t - t \tag{4}$$

or equivalently:

$$R_t = LS_t - ES_t = LF_t - EF_t \tag{5}$$

An activity is classified as critical if $R_t = 0$. The sequence of such activities forms the critical path, which determines the total project duration. Any delay in these activities will directly delay the entire project.

The time-cost trade-off (crashing) assumes a linear relationship between time and cost, expressed as:

$$\boldsymbol{Bt} = \frac{\boldsymbol{Bc} - \boldsymbol{Bn}}{\boldsymbol{Wn} - \boldsymbol{Wc}} = \frac{\Delta \boldsymbol{B}}{\Delta \boldsymbol{W}} \quad (6)$$

2.4 PERT Formulation

To accommodate uncertainty in activity durations, the PERT approach is used with three time estimates [9]. Assuming the estimates follow a Beta distribution, they are combined into a single expected value as follows:

$$t_i = \frac{a_i + 4m_i + b_i}{6} \quad (7)$$

The Beta distribution is defined as:

$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1} \quad (8)$$

or equivalently:

$$f(x) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}, 0 \le x \le 1 \quad (9)$$

where:

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 x^{a-1}(1-x)^{b-1}dx \quad (10)$$

The expected value is:

$$EX = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \cdot \frac{\Gamma(a+1)\Gamma(b)}{\Gamma(a+b+1)} = \frac{a}{a+b} \quad (11)$$

The expected square value is:

$$EX^2 = \frac{a(a+1)}{(a+b+1)(a+b)} \quad (12)$$

Thus, the variance is:

$$Var\, X = \frac{ab}{(a+b)^2(a+b+1)} \quad (13)$$

In PERT analysis, the variance of activity duration is expressed as:

$$\sigma_i^2 = \frac{(b_i - a_i)^2}{36} \quad (14)$$

and the standard deviation is:

$$\sigma_i = \sqrt{\frac{(b_i - a_i)^2}{36}} = \frac{b_i - a_i}{6} \quad (15)$$

3. Result and Discussion

3. 1 The Project Activity Data and Network Development

This study aims to analyze the optimization of time and cost in a residential construction project through the application of the Critical Path Method (CPM), crashing techniques, and the Program Evaluation and Review Technique (PERT). A quantitative approach is employed to model the project network, identify the critical path, and evaluate the probability of project completion. The study is conducted on a residential construction project located in Bandar District, Simalungun Regency, such that the results are strongly influenced by project-specific conditions, including labor costs, material prices, and technical conditions in the field.

Project data were obtained through observation and interviews with project stakeholders. Based on these data, a sequence of project activities along with their dependency relationships was established, as presented in Table 1.

Table 1. Project Activity Data

No	Type of activity	Activity symbol	Previous activities	Time (day)
1	Land preparation work	A	-	5
2	Foundation excavation work	B	A	7
3	Earth filling work	C	A	4
4	Foundation structure work	D	B, C	9
5	Wall work	E	D	8
6	Floor work	F	D	6
7	Roof truss work	G	E	10
8	Roof covering work	H	G	7
9	Electrical installation work	I	F	6
10	Ceiling work	J	H, I	5
11	Door and frame work	K	E	6
12	Window work	L	K	4

13	Clean water installation work	M	I, L	7
14	Sanitation work	N	M	5
15	Painting work	O	J, N	6
16	Finishing work	P	O	2

Based on Table 1, all project activities are systematically arranged from the preparation stage to the completion stage. Each activity has a clear dependency relationship, allowing the construction of a project network as the basis for CPM analysis.

Based on the activity data presented in Table 1, a project network was developed using the Activity-on-Node (AON) approach. The network illustrates the logical precedence relationships among all construction activities, beginning with land preparation and ending with the final finishing work. Each node represents one construction activity, while each directed arc indicates the required sequence of execution according to the project schedule.

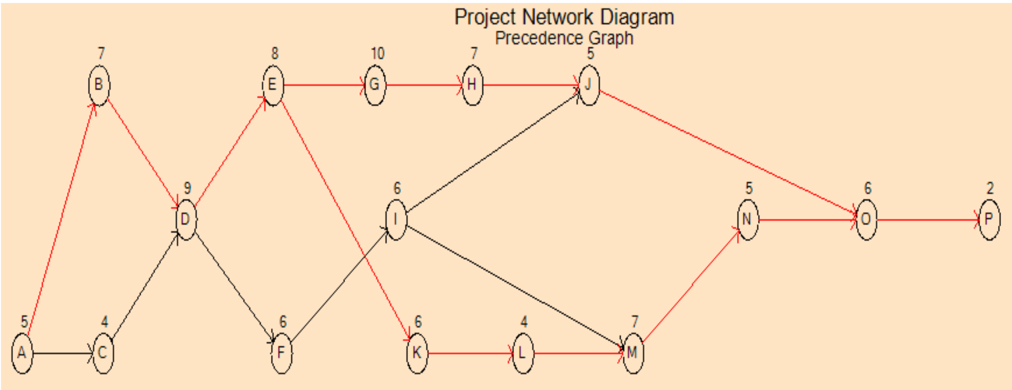

Figure 1. Project Network Diagram

As illustrated in Figure 1, the project network contains several parallel branches after the completion of the foundation structure activity (Activity D). One branch proceeds through the roofing works (Activities G and H), another follows the architectural finishing works (Activities K and L), whereas the third branch consists of electrical and utility installation activities (Activities I, M, and N). These branches subsequently converge before the painting activity (Activity O), which is followed by the final finishing activity (P). The developed network provides the basis for identifying critical activities, determining project duration using CPM, evaluating schedule acceleration through crashing analysis, and assessing project completion uncertainty using the PERT method.

3. 2 Critical Path Method Analysis

The Critical Path Method was applied to determine the earliest and latest allowable times for each project activity and to identify the sequence of activities that directly controlled the overall project duration. The forward-pass calculation was first performed to determine the Earliest Start and Earliest Finish values, followed by the backward-pass calculation to obtain the Latest Start and Latest Finish values. Total Float was subsequently calculated as the difference between the Latest Start and Earliest Start of each activity.

The CPM calculation results are presented in Table 2. The forward-pass calculation showed that the final activity was completed at day 59, indicating that the normal project duration was 59 days. Activities with a Total Float value of zero were classified as critical because any delay in these activities would directly increase the overall project completion time.

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Table 2. CPM Calculation Results for the Normal Project Schedule

Activity	Duration	Early Start (ES)	Early Finish (EF)	Late Start (LS)	Late Finish (LF)	Slack
A	5	0	5	0	5	0
B	7	5	12	5	12	0
C	4	5	9	8	12	3
D	9	12	21	12	21	0
E	8	21	29	21	29	0
F	6	21	27	27	33	6
G	10	29	39	29	39	0
H	7	39	46	39	46	0
I	6	27	33	33	39	6
J	5	46	51	46	51	0
K	6	29	35	29	35	0
L	4	35	39	35	39	0
M	7	39	46	39	46	0
N	5	46	51	46	51	0
O	6	51	57	51	57	0

P	2	57	59	57	59	0

As shown in Table 2, Activities C, F, and I were non-critical activities, with Total Float values of 3, 6, and 6 days, respectively. These float values indicate that the activities could be delayed within their allowable limits without affecting the overall project completion time. In contrast, Activities A, B, D, E, G, H, J, K, L, M, N, O, and P had zero Total Float and were therefore classified as critical activities.

The CPM analysis identified two critical paths with an equal duration of 59 days. The first critical path was A–B–D–E–G–H–J–O–P, while the second critical path was A–B–D–E–K–L–M–N–O–P. The existence of two critical paths indicates that the project schedule was controlled by two parallel activity sequences. Consequently, schedule acceleration could not be achieved merely by shortening an activity located on only one critical branch. An effective crashing strategy must either reduce the duration of an activity shared by both critical paths or simultaneously shorten activities on each critical branch.

This finding is important for the subsequent crashing analysis because reducing an activity on only one critical path would leave the second critical path unchanged. Therefore, the identification of multiple critical paths must be considered in selecting activities for project acceleration.

3.3 Project Crashing Analysis

Project crashing was performed after the CPM analysis identified two critical paths with an equal normal duration of 59 days. Because both critical paths controlled the completion time, reducing an activity located on only one critical branch would not necessarily shorten the project. Therefore, the selection of activities for acceleration considered the current critical paths, the maximum allowable duration reduction, and the incremental cost represented by the cost slope.

The cost slope expresses the additional direct cost required to shorten an activity by one day and was calculated as follows:

Cost Slope = (Crash Cost - Normal Cost) / (Normal Duration - Crash Duration)

All cost-slope values were recalculated without intermediate rounding. The corrected values are presented in Table 3. In particular, the cost slope of Activity B is IDR 50,000 per day, obtained from IDR 150,000 divided by a three-day reduction, rather than IDR 37,500 per day as reported in the previous version.

Table 3. Corrected Crashing Data and Cost-Slope Calculation

Activity	Predecessor	Normal duration	Crash duration	Normal cost (IDR)	Crash cost (IDR)	Maximum reduction	Cost slope (IDR/day)
A	-	5	2	2,200,000	2,425,000	3	75,000
B	A	7	4	3,225,000	3,375,000	3	50,000
C	A	4	2	1,800,000	1,900,000	2	50,000
D	B, C	9	4	4,000,000	4,325,000	5	65,000
E	D	8	5	3,400,000	3,700,000	3	100,000
F	D	6	4	3,200,000	3,350,000	2	75,000
G	E	10	6	4,450,000	4,650,000	4	50,000
H	G	7	5	3,075,000	3,175,000	2	50,000
I	F	6	4	2,650,000	2,750,000	2	75,000
J	H, I	5	2	1,125,000	1,425,000	3	100,000
K	E	6	3	2,700,000	2,850,000	3	50,000
L	K	4	2	1,800,000	1,900,000	2	50,000
M	I, L	7	5	2,575,000	2,675,000	2	50,000
N	M	5	2	2,200,000	2,425,000	3	75,000
O	J, N	6	3	2,700,000	2,850,000	3	50,000
P	O	2	1	950,000	950,000	1	0

Table 3 shows that Activity P had a zero cost slope because its duration could be reduced from two days to one day without increasing its direct cost. Activity B was also suitable for acceleration because it was shared by both normal critical paths and had a relatively low cost slope of IDR 50,000 per day. In contrast, shortening G, H, J, K, L, M, or N individually would affect only one of the two critical branches and would therefore not reduce the overall completion time unless a corresponding activity on the other critical branch was shortened simultaneously.

The crashing procedure was implemented iteratively. After each duration reduction, the project network was recalculated to confirm the updated completion time and critical-path structure. The target duration was set at 55 days, corresponding to a four-day reduction from the normal schedule. The detailed iterations are presented in Table 4.

Table 4. Iterative Project-Crashing Process

Stage	Activity shortened	Duration change	Cost slope (IDR/day)	Additional cost (IDR)	Cumulative cost (IDR)	Project duration (days)	Key observation
Normal schedule	-	-	-	0	0	59	Two critical paths

Iteration 1	P	2 to 1	0	0	0	58	P is common to both critical paths
Iteration 2	B	7 to 6	50,000	50,000	50,000	57	Both critical paths shortened
Iteration 3	B	6 to 5	50,000	50,000	100,000	56	Both critical paths shortened
Iteration 4	B	5 to 4	50,000	50,000	150,000	55	C becomes critical together with B

The first reduction was applied to Activity P because it shortened both critical paths by one day without additional cost. The remaining three-day reduction was obtained by shortening Activity B from seven to four days. Activity B was selected because it belonged to both normal critical paths and its full three-day crash limit incurred an additional cost of only IDR 150,000. This sequence reduced the project duration from 59 to 55 days without applying unreported acceleration to any other activity.

Following the acceleration of Activities P and B, the forward-pass and backward-pass calculations were repeated using the revised durations. The recalculated CPM parameters are shown in Table 5.

Table 5. Results of Project Schedule after Crashing

Activity	Revised duration	ES	EF	LS	LF	Total float	Status
A	5	0	5	0	5	0	Critical
B	4	5	9	5	9	0	Critical
C	4	5	9	5	9	0	Critical
D	9	9	18	9	18	0	Critical
E	8	18	26	18	26	0	Critical
F	6	18	24	24	30	6	Non-critical
G	10	26	36	26	36	0	Critical
H	7	36	43	36	43	0	Critical
I	6	24	30	30	36	6	Non-critical
J	5	43	48	43	48	0	Critical
K	6	26	32	26	32	0	Critical
L	4	32	36	32	36	0	Critical
M	7	36	43	36	43	0	Critical
N	5	43	48	43	48	0	Critical
O	6	48	54	48	54	0	Critical
P	1	54	55	54	55	0	Critical

The recalculation confirmed a final project duration of 55 days. At this stage, Activity C became critical because the completion times of B and C were both equal to day 9. The roofing branch and the door-window-water-installation branch also remained critical. Consequently, the accelerated schedule contained four critical paths: A-B-D-E-G-H-J-O-P; A-B-D-E-K-L-M-N-O-P; A-C-D-E-G-H-J-O-P; and A-C-D-E-K-L-M-N-O-P. Each path had a duration of 55 days.

The normal direct project cost was IDR 42,050,000. The three-day reduction in Activity B added IDR 150,000, while the one-day reduction in Activity P required no additional cost. Therefore, the accelerated direct cost was IDR 42,200,000. The final result represents a reduction of four days, or approximately 6.78% of the normal project duration, with an increase of approximately 0.36% in direct project cost.

These results differ from the previous calculation, which reported the acceleration of Activities B and G, an additional cost of IDR 212,500, and an unreported one-day reduction in Activity O. The revised analysis eliminates that inconsistency by explicitly documenting every crashed activity, correcting the cost slope of Activity B, and recalculating the network after each iteration.

3.3 PERT Analysis

The PERT analysis was conducted to incorporate uncertainty into the project duration estimates. For each activity, three duration estimates were used: the optimistic time (a), the most likely time (m), and the pessimistic time (b). These estimates were obtained from the project data and were retained without premature rounding.

Table 6 presents the three-time estimates and the recalculated PERT parameters. Expected durations are reported to three decimal places, while variance values are retained with sufficient precision to avoid the rounding error identified in the previous calculation.

Table 6. Three-Time Estimates and PERT Parameters for Project Activities

Activity	Type of activity	Pred.	a	m	b	Expected time, te	Std. dev., σ	Variance, σ^2
A	Land preparation work	-	3	5	6	4.833	0.500	0.250
B	Foundation excavation work	A	2	4	7	4.167	0.833	0.694
C	Earth filling work	A	2	4	5	3.833	0.500	0.250
D	Foundation structure work	B, C	3	4	6	4.167	0.500	0.250
E	Wall work	D	2	5	7	4.833	0.833	0.694
F	Floor work	D	3	6	7	5.667	0.667	0.444
G	Roof truss work	E	3	6	8	5.833	0.833	0.694
H	Roof covering work	G	4	5	7	5.167	0.500	0.250
I	Electrical installation work	F	4	6	9	6.167	0.833	0.694
J	Ceiling work	H, I	3	5	6	4.833	0.500	0.250
K	Door and frame work	E	3	6	7	5.667	0.667	0.444
L	Window work	K	2	4	5	3.833	0.500	0.250
M	Clean water installation work	I, L	4	7	9	6.833	0.833	0.694
N	Sanitation work	M	4	5	7	5.167	0.500	0.250
O	Painting work	J, N	2	3	5	3.167	0.500	0.250
P	Finishing work	O	1	1	2	1.167	0.167	0.028

The recalculated values show that the expected duration of an activity is not necessarily an integer. For example, the expected duration of Activity A is 4.833 days, rather than 5 days, and the expected duration of Activity B is 4.167 days, rather than 4 days. Preserving these decimal values throughout the network calculation prevents cumulative rounding errors and produces a more accurate estimate of the project completion time.

The expected activity durations were then substituted into the same precedence network used in the CPM analysis. A forward pass and backward pass were performed using the unrounded expected durations. The resulting schedule parameters are presented in Table 7.

Table 7. PERT Network Calculation Using Expected Activity Durations

Activity	te	ES	EF	LS	LF	Total float	Status
A	4.833	0.000	4.833	0.000	4.833	0.000	Critical
B	4.167	4.833	9.000	4.833	9.000	0.000	Critical
C	3.833	4.833	8.667	5.167	9.000	0.333	Non-critical
D	4.167	9.000	13.167	9.000	13.167	0.000	Critical
E	4.833	13.167	18.000	13.167	18.000	0.000	Critical
F	5.667	13.167	18.833	15.667	21.333	2.500	Non-critical
G	5.833	18.000	23.833	23.667	29.500	5.667	Non-critical
H	5.167	23.833	29.000	29.500	34.667	5.667	Non-critical
I	6.167	18.833	25.000	21.333	27.500	2.500	Non-critical
J	4.833	29.000	33.833	34.667	39.500	5.667	Non-critical
K	5.667	18.000	23.667	18.000	23.667	0.000	Critical
L	3.833	23.667	27.500	23.667	27.500	0.000	Critical
M	6.833	27.500	34.333	27.500	34.333	0.000	Critical
N	5.167	34.333	39.500	34.333	39.500	0.000	Critical
O	3.167	39.500	42.667	39.500	42.667	0.000	Critical
P	1.167	42.667	43.833	42.667	43.833	0.000	Critical

The forward-pass calculation produced an expected project duration of 43.833 days. The critical activities were A, B, D, E, K, L, M, N, O, and P, which formed the PERT critical path A-B-D-E-K-L-M-N-O-P. This result differs from the deterministic CPM result because PERT replaces the normal durations with weighted expected durations. In particular, the expected durations on the K-L-M-N branch were greater than those on the G-H-J branch, making the former the controlling path under uncertainty.

The project variance was obtained by summing the activity variances along the PERT critical path. Variances, rather than standard deviations, were added because the variance of the project duration is approximated as the sum of the independent activity variances on the critical path.

$$\text{Project variance} = \Sigma\sigma^2 = 3.806 \text{ days}^2$$

$$\text{Project standard deviation} = \sqrt{3.806} = 1.951 \text{ days}$$

Thus, the variance of the expected project duration was 3.806 days², while the project standard deviation was 1.951 days. These two quantities should not be interpreted interchangeably: variance is expressed in squared days, whereas standard deviation is expressed in days.

To estimate the probability of completing the project within the target duration of 49 days, the standard normal Z-score was calculated as follows:

$$Z = 2.649$$

The cumulative standard normal probability corresponding to $Z = 2.649$ is 0.9960. Therefore, the estimated probability of completing the project within 49 days is approximately 99.60%. Conversely, the estimated probability of exceeding the 49 day target is approximately 0.40%.

Table 8. Summary of the PERT Project Completion Analysis

PERT measure	Calculated value
PERT critical path	A-B-D-E-K-L-M-N-O-P
Expected project duration	43.833 days
Project variance	3.806 days²
Project standard deviation	1.951 days
Target completion time	49 days
Z-score	2.649
Probability of completion by target	99.60%
Probability of exceeding target	0.40%

The 49-day target therefore provides a substantial allowance relative to the expected PERT duration. Nevertheless, this probability is a model-based estimate that depends on the three-time estimates, activity independence, critical-path stability, and the normal approximation; it should therefore support, rather than replace, schedule monitoring and contingency planning.

Discussion

The application of the Critical Path Method (CPM), project crashing, and the Program Evaluation and Review Technique (PERT) provided a comprehensive evaluation of the residential construction project schedule [17], [18]. Unlike conventional project scheduling that focuses solely on determining project duration, the integrated analytical framework employed in this study enabled the identification of critical activities, evaluation of schedule acceleration alternatives, and assessment of project completion uncertainty. Consequently, project managers can make more informed scheduling decisions by simultaneously considering deterministic scheduling, acceleration cost, and uncertainty in activity durations [16].

The CPM analysis identified a normal project duration of 59 days and revealed that the project was controlled by two critical paths. The existence of multiple critical paths indicates that project completion depends on more than one sequence of activities having identical durations. Consequently, delays occurring in any activity located on either critical path would directly affect the overall project completion time. This finding confirms the fundamental principle of CPM that activities with zero total float require continuous monitoring throughout project execution [19].

The crashing analysis demonstrated that project duration could be reduced through selective acceleration of critical activities while minimizing additional project costs. Rather than reducing the duration of all activities, only critical activities with relatively low cost slopes were selected during the crashing process. This strategy successfully shortened the project duration while maintaining cost efficiency. The result supports previous findings reported by Gündüz et al. (2017), who concluded that cost-slope analysis provides an effective basis for selecting economically feasible acceleration alternatives. Similarly, Husen (2011) emphasized that crashing should prioritize activities located on the critical path because accelerating non-critical activities does not reduce the overall project duration.

The PERT analysis complemented the deterministic CPM results by incorporating uncertainty into activity duration estimation. Unlike CPM, which assumes fixed activity durations, PERT considers optimistic, most likely, and pessimistic estimates, thereby providing a probabilistic assessment of project completion. The relatively high probability of completing the project within the specified target duration indicates that the proposed schedule is reliable under the estimated uncertainty conditions. These findings are consistent with the work of Moder et al. (1983), who demonstrated that PERT is particularly useful for evaluating schedule uncertainty and supporting managerial decision-making under uncertain project environments.

The integration of CPM, crashing analysis, and PERT provides several advantages compared with the application of each technique independently. CPM identifies activities that determine project duration, crashing determines the most economical acceleration strategy, whereas PERT evaluates the likelihood of completing the project within a specified deadline. Therefore, integrating these three techniques provides a more comprehensive decision-support framework for balancing project duration, project cost, and schedule uncertainty. Similar integrated approaches have been recommended in construction project management because they improve planning accuracy and facilitate more effective project control [16].

Compared with previous studies, the present research offers a more comprehensive scheduling evaluation by integrating deterministic scheduling, schedule acceleration, and probabilistic time analysis within a single analytical framework. While many earlier studies focused only on CPM or PERT separately, this study demonstrates that combining the three methods provides richer managerial information for construction scheduling decisions. Specifically, the proposed framework enables project managers not only to identify critical activities but also to evaluate cost-effective acceleration strategies and quantify the probability of meeting contractual completion deadlines.

From a practical perspective, the proposed analytical framework can assist contractors, consultants, and project managers in improving scheduling decisions during both project planning and execution. By integrating CPM, crashing analysis, and PERT, project stakeholders can identify activities requiring close supervision, evaluate economically feasible acceleration alternatives, and estimate project completion risks before project implementation begins. Consequently, more effective resource allocation, schedule control, and risk mitigation strategies can be developed, ultimately improving overall project performance.

4. Conclusion

This The application of the Critical Path Method (CPM), project crashing, and the Program Evaluation and Review Technique (PERT) provided a comprehensive evaluation of the residential construction project schedule [17], [18]. Unlike conventional project scheduling that focuses solely on determining project duration, the integrated analytical framework employed in this study enabled the identification of critical activities, evaluation of schedule acceleration alternatives, and assessment of project completion uncertainty. Consequently, project managers can make more informed scheduling decisions by simultaneously considering deterministic scheduling, acceleration cost, and uncertainty in activity durations [16].

The CPM analysis identified a normal project duration of 59 days and revealed that the project was controlled by two critical paths. The existence of multiple critical paths indicates that project completion depends on more than one sequence of activities having identical durations. Consequently, delays occurring in any activity located on either critical path would directly affect the overall project completion time. This finding confirms that activities with zero total float require continuous monitoring throughout project execution to prevent delays in project completion [19].

The crashing analysis demonstrated that project duration could be reduced through selective acceleration of critical activities while minimizing additional project costs. Rather than reducing the duration of all activities, only critical activities with relatively low cost slopes were selected during the crashing process. This strategy successfully shortened the project duration while maintaining cost efficiency. These findings support previous studies showing that cost-slope analysis is an effective approach for selecting economically feasible acceleration alternatives and that schedule acceleration should focus primarily on activities located on the critical path [17], [18].

The PERT analysis complemented the deterministic CPM results by incorporating uncertainty into activity duration estimation. Unlike CPM, which assumes fixed activity durations, PERT considers optimistic, most likely, and pessimistic estimates, thereby providing a probabilistic assessment of project completion. The relatively high probability of completing the project within the specified target duration indicates that the proposed schedule is reliable under the estimated uncertainty conditions. This finding is consistent with previous studies demonstrating that PERT is an effective technique for evaluating schedule uncertainty and supporting project planning under uncertain conditions [16].

The integration of CPM, crashing analysis, and PERT provides several advantages compared with the application of each technique independently. CPM identifies activities that determine project duration, crashing determines the most economical acceleration strategy, whereas PERT evaluates the likelihood of completing the project within a specified deadline. Therefore, integrating these three techniques provides a more comprehensive decision-support framework for balancing project duration, project cost, and schedule uncertainty. Similar integrated approaches have been recommended because they improve planning accuracy and facilitate more effective project control [17].

Compared with previous studies, the present research offers a more comprehensive scheduling evaluation by integrating deterministic scheduling, schedule acceleration, and probabilistic time analysis within a single analytical framework. While many previous studies applied CPM, crashing, or PERT individually, this study demonstrates that integrating the three methods provides more comprehensive information for construction scheduling decisions by simultaneously considering project duration, acceleration cost, and completion uncertainty [16].

From a practical perspective, the proposed analytical framework can assist contractors, consultants, and project managers in improving scheduling decisions during both project planning and execution. By integrating CPM, crashing analysis, and PERT, project stakeholders can identify activities requiring close supervision, evaluate economically feasible acceleration alternatives, and estimate project completion risks before project implementation begins. Consequently, more effective resource allocation, schedule control, and risk mitigation strategies can be developed, ultimately improving overall project performance.

References

[1] M. Aghileh, A. Tereso, and F. Alvelos, “Multi-Project Scheduling with Uncertainty and Resource Flexibility : A Narrative Review and Exploration of Future Landscapes,” 2025.

[2] A. Francis and D. Ph, “Simulating Uncertainties in Construction Projects with Chronographical Scheduling Logic,” vol. 143, no. 1, pp. 1–14, 2017, doi: 10.1061/(ASCE)CO.1943-7862.0001212.

[3] M. Aghileh, A. Tereso, F. Alvelos, and M. O. Monteiro, “Multi-project scheduling under uncertainty and resource flexibility : a systematic literature review ARTICLE HISTORY,” Prod. Manuf. Res., vol. 12, no. 1, 2024, doi: 10.1080/21693277.2024.2319574.

[4] G. R. Putra, R. Ariftiarno, L. Karlitasari, and D. M. Akhmad, “Faculty Information Systems Project Management : A Comparison of CPM And PERT Methods,” vol. 22, no. January, pp. 12–20, 2025.

[5] K. B. Bagshaw, “PERT and CPM in Project Management with Practical Examples,” pp. 215–226, 2021, doi: 10.4236/ajor.2021.114013.

[6] K. B. Bagshaw, “NEW PERT and CPM in Project Management with Practical Examples,” pp. 215–226, 2021, doi: 10.4236/ajor.2021.114013.

[7] J. Song, A. Martens, and M. Vanhoucke, “Using Earned Value Management and Schedule Risk Analysis with resource constraints for project control,” Eur. J. Oper. Res., 2021, doi: 10.1016/j.ejor.2021.05.036.

[8] A. Fadjar, A. A. Ali, A. Setiawan, C. Engineering, D. Study, and S. Tengah, “Estimating House Construction Completion Time With Probabilistic Activities Duration Using The Program Evaluation And Review Technique (PERT) Method,” J. Sains dan Teknol. Tadulako, vol. 9, no. 1, pp. 1–9, 2023.

[9] S. Automatisés, M. A. Jouda, and M. A. K. Shiker, “Comparing the Effectiveness of PERT and CPM Techniques in a House Construction Project : A Case Study,” vol. 57, no. 3, pp. 781–785, 2024.

[10] A. M. Uktolseja, M. Wullur, and M. M. Karuntu, “Evaluasi Pelaksanaan Proyek Menggunakan Metode PERT Dan CPM (Studi Kasus : Preservasi Jalan Tolango-Paguyaman , Tolango-Bulontio) Evaluation Of Project Implementation Using PERT And CPM Methods (Case Study : Tolango – Paguyaman , Tolango – Bulontio Roa,” vol. 6, no. 2, pp. 1079–1090, 2023.

[11] R. A. Husna, N. F. Ilmiyah, and N. C. Resti, “Implementasi CPM dan PERT dalam Memprediksi Durasi serta Biaya Pembangunan Musala Al-Ikhlas di Kotawaringin Barat,” J. Focus Action Res. Math. (Factor M), vol. 5, no. 1, pp. 97–109, 2022, doi: 10.30762/f_m.v5i1.633.

[12] A. Fausan and W. Sulistiono, “Evaluasi Manajemen Waktu Dan Biaya Proyek Menggunakan Metode Critical Path Method Studi Kasus Pekerjaan Pembangunan Ruang Kelas SDN 22 Belopa Kabupaten Luwu,” vol. 25, pp. 834–847, 2025.

[13] I. M. Leimena et al., “PERT Pada Proyek Instalasi Pipa Dan Mesin Pompa,” vol. 11, no. 1, 2024.

[14] D. Lima, L. Trentini, S. Da Silva, C. Eduardo, and S. Turrioni, “01-059 – PERT and CPM in construction : analysis of benefits , limitations and future trends – PERT y CPM en la construcción : análisis de ventajas , limitaciones y tendencias futuras,” no. July, pp. 466–480, 2025.

[15] M. Vanhoucke, “▶ Integrated Project Management and Control: First Comes the Theory, Then the Practice,” Cham Springer Int. Publ., vol. 49, no. 0, p. 6221, 2014.

[16] B. Heizer, Jay;Render and G. Edition, “Operations Management : Sustainability and Supply Chain Management,” 2020.

[17] M. Haziq, F. Abdul, S. Noor, and A. Mohd, “EKST Optimization of Chicken Farm Renovation Project Using PERT , CPM technique and POM-QM software,” vol. 4, no. 1, pp. 183–193, 2024.

[18] B. N. Kurniawan and N. B. Mulyono, “Minimizing Delay in Construction Project at PT Freeport Indonesia using Crashing CPM-PERT Approach and Monte Carlo Simulation Keywords :,” vol. 9, no. 5, pp. 61–69, 2024.

[19] J. Yang, “Reviewing Construction Schedule Float Management,” Open Constr. Build. Technol. J., vol. 11, no. 300, pp. 1–13, 2017, doi: 10.2174/1874836801711010001.